# Updated 2021 assessments of Jasus tristani rock lobster at Gough island 

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## Summary

This paper provides an updated assessment of the Jasus tristani rock lobster resource at Gough island. This assessment includes updated data from the commercial fishery and biomass surveys for the 2018 and 2019 seasons. Data from the 2020 season are not included as they are not yet fully available. This assessment was last updated in 2018. The updated 2021 assessment has produced somewhat more optimistic results with respect to current spawning biomass. Current resource abundance is estimated to be $88 \%$ of pristine - a very healthy state. This updated assessment model will function as the underlying baseline operating model in the development of new 2021 OMP.

KEY WORDS: Gough island, Jasus tristani, stock assessment

## Introduction

The age-structured population model used for this assessment is described fully in Johnston and Butterworth (2013). The assessment was last updated in 2018 (Johnston and Butterworth 2018). The updated 2021 assessment includes the following data:

1) Standardised longline CPUE data for 1997-2019 (Johnston 2020). Note this GLMM takes into account length of fishing trip information.
2) Biomass survey CPUE data (2006-2019, with data for 2008 absent because there was no survey that year).
3) Catch-at-length data from the onboard observers (males and females separate) (19972019).
4) Catch-at-length data from the biomass survey (males and females separate) (2006-2019, with 2008 data absent).
5) Discard \% (2003-2019; earlier data are not included in the likelihood due to their unreliability).

Data to 2017 only were available for the previous 2018 assessment. Data from the 2020 season are not included here as they are not yet fully available. A new modification to the 2020 assessment is to omit pre-2003 Discard \% data due to the fact that these data are now considered questionable as they show little difference from subsequent data despite a 5 mm increase in minimum size at that time.

## Value of $\mathrm{F}_{2009}$

The previous 2018 assessment assumed a value of fishing proportion in 2009, $\mathrm{F}_{2009}$, of 0.30 . Here the appropriateness of this value is re-examined. Figure 1a plots the total -InL values for a range of $\mathrm{F}_{2009}$. From this plot it is clear that assuming an $F_{2009}=0.2$ is the more appropriate value (provides the best fit to the data). Hence the 2021 RC model will assume and $\mathrm{F}_{2009}=0.2$, although sensitivity to this value will continue to be explored.

## Value of natural mortality $\boldsymbol{M}$

Table 1 lists the values of natural mortality estimates for lobster species around the world obtained from the RAM legacy database. The average value is $M=0.17$. For the South African west coast rock lobster Jasus lalandii, adult $M$ is assumed to be 0.10 . The previous 2018 assessment assumed a value of $M=0.2$. Here the appropriateness of this value is re-examined. Figure 1 b plots the total - InL values for a range of $M$ values (ranging from 0.10 to 0.20 as guided by the results presented in Table 1). From this plot it is clear that assuming an $M=0.1$ is the more appropriate value (provides the best fit to the data). Hence the 2021 RC model will assume $M=0.1$, although sensitivity to this value will continue to be explored.

## Sensitivity models

Results are initially run for the same set of assumptions assumed in 2018. Table 1 reports these Reference case model assumptions. A series of sensitivity models are then run to explore the sensitivity of the assessment results to these assumptions. These are:

Sen1: fix $h=0.90$
Sen2: fix $h=0.80$

Sen3: fix $h=0.70$

Sen3b: fox $h=0.50$
Sen4: $M=0.2$
Sen5: $d=0.2$
Sen6: $F_{2009}=0.3$

## Results

## RC model fits

Table 3a reports the Gough 2021 updated RC assessment results, and provides the 2018 assessment results for comparison. Removing the 1997-2002 Discard \% data from the likelihood has improved fits to

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the CPUE and CAL data. The model fit to the commercial CPUE data is particularly good for recent years (Figure 2).

Figure 3 shows the estimated selectivity functions for both the commercial and biomass survey gears. Figures 4 a and b show the time-varying selectivity values of the estimated parameters $\mu$ and $P$.

Figures 5 a and b show the average fits to the catch-at-length data for males and females for both the commercial and biomass survey data.

Figures 6 a and b show the standardised CAL residuals for the commercial and biomass survey data. The dark bubbles reflect positive and the light bubbles negative residuals, with the bubble radii proportional to the magnitudes of the residuals.

## Results of sensitivity model fits

Table 3b reports results for the sensitivity models.

## Discussion

The current $\mathrm{Bsp} / \mathrm{K}$ is estimated to be healthy at 0.88 . Compared to the 2018 assessment results, the updated assessment is more optimistic in terms of current spawning biomass. This is a result of a number of factors: fitting to further data; estimation of a new female selectivity parameter, removal of pre-2003 Discard \% data from the likelihood, and changing the RC $M$ assumption from 0.2 to $0.1 \mathrm{yr}^{-1}$ and the $F_{2009}$ assumption of 0.3 to 0.2 following initial model fits indicating that these changes were appropriate (as they resulted in better fits to the data). The new 2021 OMP development will explore sensitivity to the various sensitivity tests.

## References

Johnston, S.J. 2020. Further GLM analyses of Nightingale, Inaccessible and Gough CPUE data to incorporate trip length data. MARAM/Tristan/2020/Nov/15. 8pp.

Johnston, S.J. and Butterworth, D.S. 2013. The age structured population modeling approach for the assessment of the rock lobster resources at the Tristan da Cunha group of islands. MARAM/Tristan/2013/Mar/07. 15pp.

Johnston, S.J. and Butterworth, D.S. 2018. Updated 2018 rock lobster assessments for Inaccessible and Gough islands. MARAM/Tristan/2018/JUL/08.

Table 1: Natural mortality estimates obtained from the RAM legacy database.

| Lobster stock | $\mathbf{M ~ y r}^{\mathbf{- 1}}$ |
| :--- | :---: |
| American lobster Georges Bank | 0.150 |
| American lobster Gulf of Maine | 0.150 |
| American lobster Southern New England | 0.150 |
| Yellow squat lobster Central-Southern Chile | 0.300 |
| Yellow squat lobster Northern Chile | 0.300 |
| Red squat lobster Central-Southern Chile | 0.350 |
| Red squat lobster Northern Chile | 0.350 |
| West coast rock lobster South Africa Areas 1-2 | 0.110 |
| West coast rock lobster South Africa Areas 3-4 | 0.110 |
| West coast rock lobster South Africa Areas 5-6 | 0.110 |
| West coast rock lobster South Africa Area 7 | 0.110 |
| West coast rock lobster South Africa Area 8 | 0.110 |
| Southern spiny lobster South Africa South |  |
| coast | 0.100 |
| Red rock lobster New Zealand Area CRA1 | 0.125 |
| Red rock lobster New Zealand Area CRA2 | 0.161 |
| Red rock lobster New Zealand Area CRA3 | 0.251 |
| Red rock lobster New Zealand Area CRA4 | 0.322 |
| Red rock lobster New Zealand Area CRA5 | 0.132 |
| Red rock lobster New Zealand Area CRA7 | 0.103 |
| Red rock lobster New Zealand Area CRA8 | 0.095 |
| Rock lobster South Australia Northern Zone | 0.100 |
| Rock lobster South Australia Southern Zone | 0.100 |
|  | 0.171 |
| Average |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

Table 2: Reference case model assumptions.

|  | $\mathbf{2 0 1 8}$ <br> assessment | $\mathbf{2 0 2 1}$ <br> assessment |
| :--- | :---: | :---: |
| $M$ natural mortality | 0.2 | 0.1 |
| Mean of the prior on $h$ <br> (the SR steepness parameter) | 0.95 | 0.95 |
| $d$ (discard mortality rate) | 0.1 | 0.1 |
| $F(2009)$ <br> (harvest proportion in 2009) | 0.3 | 0.2 |

Table 3a: Gough updated 2021 assessment results for the Reference Case (RC) model. The 2018 assessment results are reported in the first column to allow comparison. The shaded values are fixed on input.

|  | 2018 RC assessment | $2021 \text { RC }$ assessment |
| :---: | :---: | :---: |
| \# parameters | 105 | 113 |
| $K$ | 302 | 331 |
| $h$ | 0.87 | 0.86 |
| M | 0.2 | 0.1 |
| $d$ (discard mortality rate) | 0.1 | 0.1 |
| $\sigma_{\text {length }}$ | 0.2 | 0.2 |
| $\mathrm{F}_{2009}$ fixed at | 0.3 | 0.2 |
| $\theta$ | 0.616 | 0.603 |
| Bsp(1990)/Ksp | 0.58 | 0.56 |
| Bsp(2018)/Ksp | 0.77 | 0.87 |
| Bsp(2020)/Ksp | - | 0.88 |
| $B \exp (2017)$ $(\operatorname{Bexp}(2017) / \operatorname{Bexp}(1990))$ | $\begin{gathered} 134 \\ (0.93) \\ \hline \end{gathered}$ | $\begin{gathered} 166 \\ (0.98) \\ \hline \end{gathered}$ |
| $B \exp (2019)$ $(\operatorname{Bexp}(2019) / \operatorname{Bexp}(1990))$ | - | $\begin{gathered} \hline 203 \\ (1.20) \end{gathered}$ |
| Programs | Gough18.tpl | Gough21y.tpl |

Table 3b: Gough 2021 assessment sensitivity model results. Fixed parameter values are in shaded block. Values in red are those altered from the RC.

|  | RC | Sen1 <br> Fix $\boldsymbol{h}=\mathbf{0 . 9 0}$ | Sen2 <br> Fix $\boldsymbol{h}=\mathbf{0 . 8 0}$ | Sen3 <br> Fix $\boldsymbol{h}=\mathbf{0 . 7 0}$ | $\begin{gathered} \text { Sen3b } \\ \text { Fix } h=0.50 \end{gathered}$ | $\begin{gathered} \text { Sen4 } \\ M=0.2 \end{gathered}$ | $\begin{aligned} & \text { Sen5 } \\ & d=0.2 \end{aligned}$ | $\begin{gathered} \text { Sen6 } \\ F_{2009}=0.3 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | 331 | 313 | 322 | 335 | 397 | 363 | 286 | 331 |
| $h$ | 0.86 | Fix=0.90 | Fix=0.80 | Fix $=0.70$ | Fix $=0.50$ | 0.87 | 0.88 | 0.85 |
| M | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.2 | 0.1 | 0.1 |
| $h$ prior mean | 0.95 | - | - | - | - | 0.95 | 0.95 | 0.95 |
| $d$ (discard mortality rate) | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.2 | 0.1 |
| $\mathrm{F}_{2009}$ fixed at | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.3 |
| $\theta$ | 0.603 | 0.629 | 0.597 | 0.567 | 0.478 | 0.619 | 0.656 | 0.591 |
| -InL total | 11.79 | 15.78 | 15.89 | 16.08 | 17.44 | 16.74 | 18.43 | 17.09 |
| -InL CPUE T | -21.06 | -21.00 | -20.97 | -21.08 | -21.07 | -21.26 | -18.37 | -21.02 |
| -InL CPUE longline | $\begin{gathered} \hline-11.74 \\ (0.0001) \\ \hline \end{gathered}$ | $\begin{gathered} -11.75 \\ (0.0001) \end{gathered}$ | $\begin{gathered} -11.74 \\ (0.0003) \end{gathered}$ | $\begin{gathered} -11.75 \\ (0.0001) \end{gathered}$ | $\begin{gathered} -11.75 \\ (0.0001) \end{gathered}$ | $\begin{gathered} -11.75 \\ (0.0001) \end{gathered}$ | $\begin{gathered} -9.13 \\ (0.064) \end{gathered}$ | $\begin{gathered} \hline-11.75 \\ (0.0001) \end{gathered}$ |
| -InL CPUE Survey Leg1 | -9.31 (0.282) | -9.25 (0.284) | -9.23 (0.284) | -9.34 (0.281) | -9.32 (0.281) | -9.51 (0.277) | $\begin{gathered} -9.24 \\ (0.284) \end{gathered}$ | -9.27 (0.283) |
| -InL CAL T | 227.94 | 269.05 | 272.87 | 270.77 | 271.48 | 275.14 | 292.07 | 296.23 |
| -InL CAL onboard observer | $\begin{aligned} & 351.04 \\ & (0.140) \end{aligned}$ | $\begin{aligned} & \hline 392.76 \\ & (0.152) \end{aligned}$ | $\begin{aligned} & \hline 396.76 \\ & (0.153) \end{aligned}$ | $\begin{aligned} & \hline 394.43 \\ & (0.152) \end{aligned}$ | $\begin{aligned} & 395.02 \\ & (0.152) \end{aligned}$ | $\begin{aligned} & 399.10 \\ & (0.153) \end{aligned}$ | $\begin{aligned} & \hline 416.05 \\ & (0.158) \end{aligned}$ | $\begin{aligned} & \hline 419.70 \\ & (0.160) \end{aligned}$ |
| -InL CAL Survey Leg 1 | $\begin{aligned} & -123.10 \\ & (0.076) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline-123.70 \\ & (0.075) \end{aligned}$ | $\begin{aligned} & \hline-123.89 \\ & (0.074) \end{aligned}$ | $\begin{aligned} & \hline-123.66 \\ & (0.075) \end{aligned}$ | $\begin{gathered} -123.55 \\ (0.076) \end{gathered}$ | $\begin{aligned} & \hline-123.96 \\ & (0.075) \end{aligned}$ | $\begin{aligned} & \hline-123.98 \\ & (0.075) \\ & \hline \end{aligned}$ | $\begin{aligned} & -123.48 \\ & (0.075) \end{aligned}$ |
| SR1 pen | 3.63 | 3.46 | 3.21 | 3.39 | 2.87 | 3.94 | 3.12 | 3.36 |
| -InL discard | 1.45 | 2.82 | 2.76 | 1.40 | 2.77 | 2.63 | 1.64 | 1.61 |
| Bsp(1990)/Ksp | 0.56 | 0.58 | 0.59 | 0.56 | 0.45 | 0.58 | 0.61 | 0.55 |
| Bsp(2018)/Ksp | 0.87 | 0.84 | 0.89 | 0.88 | 0.73 | 0.89 | 0.84 | 0.86 |
| Bsp(2020)/Ksp | 0.88 | 0.84 | 0.87 | 0.86 | 0.75 | 0.87 | 0.83 | 0.85 |
| $\begin{gathered} B \exp (2017) \\ (\operatorname{Bexp}(2017) / \exp (1990)) \end{gathered}$ | $\begin{gathered} \hline 166 \\ (0.98) \end{gathered}$ | $\begin{gathered} 164 \\ (0.92) \end{gathered}$ | $\begin{gathered} 164 \\ (0.95) \end{gathered}$ | $\begin{gathered} 165 \\ (0.96) \end{gathered}$ | $\begin{gathered} 166 \\ (0.99) \end{gathered}$ | $\begin{gathered} \hline 166 \\ (1.03) \end{gathered}$ | $\begin{gathered} \hline 130 \\ (0.78) \end{gathered}$ | $\begin{gathered} 166 \\ (0.97) \end{gathered}$ |
| $\begin{gathered} B \exp (2019) \\ (B \exp (2019) / B \exp (1990)) \end{gathered}$ | $\begin{gathered} 203 \\ (1.20) \end{gathered}$ | $\begin{gathered} 203 \\ (1.13) \end{gathered}$ | $\begin{gathered} 201 \\ (1.16) \end{gathered}$ | $\begin{gathered} 203 \\ (1.18) \end{gathered}$ | $\begin{gathered} 203 \\ (1.22) \end{gathered}$ | $\begin{gathered} 204 \\ (1.26) \end{gathered}$ | $\begin{gathered} 159 \\ (0.96) \end{gathered}$ | $\begin{gathered} 218 \\ (1.27) \end{gathered}$ |
| Programs | Gough21y.tpl | gS1.tpl | gS2.tpl | gS3.tpl | gS3b.tpl | gS4.tpl | gS5.tpl | gS6.tpl |

Figure 1a: - InL (total) for a range of fixed $\mathrm{F}_{2009}$ values. The red vertical line indicates best fit to the data (lowest -InL value). [Here $M=0.2$ ]


Figure 1b: -InL (total) for a range of natural mortality $M$ values. The red vertical line indicates best fit to the data (lowest -InL value amongst the $F_{2009}$ values considered). [Here $F_{2009}=0.2$ ]


Figure 2: Gough 2021 RC assessment results. The green dashed lines indicate the 2018 assessment's estimated values.


Figure 3: Gough RC assessment selectivity functions.


Figure 4a: Gough RC assessment estimated $\mu$ residuals (used for selectivity function variability).


Figure 4b: Gough RC assessment estimated $P$ residuals (used for female selectivity function variability).


Figure 5a: Gough RC assessment commercial longline CAL fits averaged over years.


Figure 5b: Gough RC assessment biomass survey CAL fits averaged over years.



Figure 6a: Gough RC assessment standardised commercial longline CAL residuals. The dark bubbles reflect positive and the light bubbles negative residuals, with the bubble radii proportional to the magnitudes of the residuals.


Figure 6b: Gough RC assessment standardised biomass survey Leg1 CAL residuals. The dark bubbles reflect positive and the light bubbles negative residuals, with the bubble radii proportional to the magnitudes of the residuals.


Biomass survey female CAL residuals


## Appendix: The Age-structured Production Modelling approach for assessment of the Jasus tristani Rock Lobster Resources at the Tristan da Cunha group of islands

The stock assessment approach for all four islands of the Tristan da Cunha group is to use an agestructured production model (ASPM) to fit to catch, longline standardised CPUE and catch-at-length (CAL) data, as well as biomass survey indices and their CAL data. The models consider catches from only 1990, i.e. models are initiated in 1990. The method for setting up the initial population age structure in 1990 is given below.

## 1. The population model

The resource dynamics are modeled by the equations:

$$
\begin{align*}
& N_{y+1,0}^{m}=R_{y+1}  \tag{1}\\
& N_{y+1,0}^{f}=R_{y+1} \tag{2}
\end{align*}
$$

$$
\begin{align*}
& N_{y+1, a+1}^{m}=\sum_{l}\left[\vec{N}_{y, a, l}^{m} e^{-M^{m} / 2}-\vec{C}_{y, a, l}^{m}-D_{y, a, l}^{m}\right] e^{-M_{a} / 2}  \tag{3}\\
& N_{y+1, a+1}^{f}=\sum_{l}\left[\vec{N}_{y, a, l}^{f} e^{-M^{f} / 2}-\vec{C}_{y, a, l}^{f}-D_{y, a, l}^{f}\right] e^{-M_{a} / 2} \tag{4}
\end{align*}
$$

$N_{y+1, p}^{m}=\sum_{l}\left[\vec{N}_{y, p-1, l}^{m} e^{-M_{a} / 2}-\vec{C}_{y, p-1, l}^{m}-D_{y, p-1, l}^{m}\right] e^{-M_{a} / 2}+\sum_{l}\left[\vec{N}_{y, p, l}^{m} e^{-M_{a} / 2}-\vec{C}_{y, p, l}^{m}-D_{y, p, ;}^{m}\right] e^{-M_{a} / 2}$
$N_{y+1, p}^{f}=\sum_{l}\left[\vec{N}_{y, p-1, l}^{f} e^{-M_{a} / 2}-\vec{C}_{y, p-1, l}^{f}-D_{y, p-1, l}^{f}\right] e^{-M_{a} / 2}+\sum_{l}\left[\vec{N}_{y, p, l}^{f} e^{-M_{a} / 2}-\vec{C}_{y, p, l}^{f}-D_{y, p, l}^{f}\right] e^{-M_{a} / 2}$
where
$N_{y, a}^{m / f} \quad$ is the number of male or female $(m / f)$ lobsters of age $a$ at the start of year $y$,
$\vec{N}_{y, a, l}^{m / f} \quad$ is the number of male or female $(m / f)$ lobsters of age $a$ of length $/$ at the start of year $y$ (see equation 15 ),
$M_{a} \quad$ denotes the natural mortality rate for male and female lobsters aged $a$ years (and here identical for male and female lobsters). Note that for the Reference Case Operating Model this value is fixed at 0.10 for ages 0 to 9 , and increased to a value of 1.5 for ages $a=10+$. Alternate values of $M$ for lobsters aged $a=10+$ are explored in robustness tests.
$\vec{C}_{y, a, l}^{m / f} \quad$ is the catch of male or female ( $m / f$ ) lobsters of age $a$ of length / in year $y$,
$D_{y, a, l}^{m / f} \quad$ is the number of male or female $(\mathrm{m} / \mathrm{f})$ lobsters of age $a$ of length / in year $y$ that die due to discard mortality, and
$p$ is the maximum age considered (taken to be a plus-group, and set equal to 20 here).

The number of recruits of age 0 , of each sex, at the start of year $y$ is related to the spawner stock size by a stock-recruitment relationship:

$$
\begin{equation*}
R_{y}=\frac{\alpha B_{y}^{s p}}{\beta+\left(B_{y}^{s p}\right)^{\gamma}} e^{\varsigma_{y}-\sigma_{R}^{2} / 2} \tag{7}
\end{equation*}
$$

where
$\alpha, \beta$ and $\gamma$ are spawner biomass-recruitment parameters ( $\gamma=1$ for a Beverton-Holt relationship),
$\varsigma_{y}$ reflects fluctuation about the expected (mean) recruitment for year $y$ (here we estimate stock-recruit residuals for the period 1992-2016) where $\varsigma_{y} \sim N\left(0, \sigma_{R}^{2}\right)$ with $\sigma_{R}=0.4$ and
$B_{y}^{s p}$ is the spawner biomass at the start of year $y$, given by:

$$
\begin{align*}
& B_{y}^{s p}=\sum_{a=1}^{p} \sum_{l=1}^{180}\left[f_{l} Q_{a, l}^{f} w_{l}^{f} N_{y, a}^{f}\right]  \tag{8a}\\
& B_{y}^{s p}=\sum_{a=1}^{p} N_{y, a}^{f} \sum_{l=1}^{180}\left[f_{l} Q_{a, l}^{f} w_{l}^{f}\right]  \tag{8b}\\
& B_{y}^{s p}=\sum_{a=1}^{p}\left[N_{y, a}^{f} X_{a}\right] \tag{8c}
\end{align*}
$$

where

$$
\begin{equation*}
X_{a}=\sum_{l=1}^{180}\left[f_{l} \quad Q_{a, l}^{f} w_{l}^{f}\right] \tag{8d}
\end{equation*}
$$

where $w_{l}^{f}$ is the mass of female lobsters at length $l$, and $f_{l}$ is the proportion of lobster of length $l$ that are mature.

## OLIVA effects

For Inaccessible it is assumed that there is a once off $35 \%$ extra mortality event in 2011 for all aged lobsters. For Nightingale impact that the OLIVA had on the resource at Nightingale was initially assumed unchanged from the 2015 assessment and assumes the following:
i) an $80 \%$ once off additional mortality of juvenile lobsters aged 1,2 and 3 years during the 2011 season, and
ii) a $0 \%$ once off additional mortality on adults (ages 4+) during the 2011 season (as assumed for the 2017 assessments, whereas a value of $50 \%$ was used for the 2014 and 2015 RC models).

The 80\% juvenile/50\% adult mortality assumptions were initially considered reasonable for the 2014 and 2015 assessments ${ }^{1}$, but more recent CPUE data (since 2013) indicate that it is very unlikely that there was much if any impact on the adults as a result of the OLIVA incident - hence the modification to assume a $0 \%$ once off additional mortality on adults.

Results of the updated 2020 assessments suggest that the $80 \%$ once off additional juvenile mortality is now unlikely to be the most probable scenario.

## Maturity at length

Pollock (1991) produced plots of the proportion of female lobsters mature for different carapace lengths at Inaccessible and Nightingale islands. Here we assume that the results for Inaccessible island are likely to be similar to those at Tristan. Using Pollock's values, the function below in Figure A1 is assumed to apply for lobsters at all four islands:


Figure A1: Proportion of females mature versus length.

[^0]In order to work with estimable parameters that are more meaningful biologically, the stock-recruit relationship is re-parameterised in terms of the pre-exploitation equilibrium female spawning biomass, $K^{s p}$, and the "steepness" of the stock-recruit relationship (recruitment at $B^{s p}=0.2 K^{s p}$ as a fraction of recruitment at $\left.B^{s p}=K^{s p}\right)$ :

$$
\begin{equation*}
\alpha=\frac{4 h R_{1}}{5 h-1} \tag{9}
\end{equation*}
$$

and

$$
\begin{equation*}
\beta=\frac{\left(K^{s p}(1-h)\right)}{5 h-1} \tag{10}
\end{equation*}
$$

where

$$
\begin{equation*}
R_{1}=K^{s p} /\left[\sum_{a=1}^{p-1} X_{a} e^{-\sum_{a=0}^{a-1} M_{a^{\prime}}}+X_{p} \frac{e^{-\sum_{a=0}^{p-1} M_{a^{\prime}}}}{1-e^{-M_{p}}}\right] \tag{11}
\end{equation*}
$$

The total catch by mass in year $y$ is given by:

$$
\begin{equation*}
C_{y}=\sum_{m / f} \sum_{a} \sum_{l \geq \min } w_{l}^{m / f} \vec{C}_{y, a, l}^{m / f} \tag{12}
\end{equation*}
$$

where

$$
\begin{align*}
& \vec{C}_{y, a, l}^{m}=\vec{N}_{y, a l}^{m} I_{y, l}^{m, \text { comm }} F_{y}  \tag{13}\\
& \vec{C}_{y, a, l}^{f}=\vec{N}_{y, a, l}^{f} S_{y, l}^{f, c o m m} F_{y} \tag{14}
\end{align*}
$$

where $w_{l}^{m / f}$ denotes the mass of a $m / f$ lobster of length $l$, and where
$S_{y, l}^{m / f, g e a r} \quad$ is the length-specific selectivity for male/female lobsters in year y for a given gear type (either commercial or survey),
$F_{y} \quad$ is the fishing proportion in year $y$ for lobsters, and which is constrained to be
$\leq 0.90$, and where

$$
F_{y}=\frac{C_{y}^{o b s}}{\sum_{a} \sum_{l \geq \min }\left[w_{l}^{m} \vec{N}_{y, a, l}^{m} S_{y, l}^{m, c o m m} e^{-\frac{M_{a}}{2}}\right]\left[w_{l}^{f} \vec{N}_{y, a, l}^{f} S_{y, l}^{f, c o m m} e^{-\frac{M_{a}}{2}}\right]}
$$

$\min \quad$ is the minimum legal carapace length in mm , and

$$
\begin{equation*}
\vec{N}_{y, a, l}^{m / f}=N_{y, a}^{m / f} Q_{a, t}^{m / f} \tag{15}
\end{equation*}
$$

where $Q_{a, l}^{m / f}$ is the proportion of fish of age $a$ that fall in the length group $I$ for the sex and area concerned (thus $\sum_{l} Q_{a, l}^{m / f}=1$ for all ages $a$ ).
The matrix $Q$ is calculated under the assumption that length-at-age is normally distributed about a mean given by the von Bertalanffy equation, i.e.:

$$
\begin{equation*}
l_{a} \sim N^{n}\left[\left[_{o}^{m \prime \prime}\left(1-e^{-\pi(a-a s)}\right) ; \theta_{a}^{2}\right]\right. \tag{16}
\end{equation*}
$$

where
$N^{*} \quad$ is the normal distribution truncated at $\pm 3$ standard deviations, and
$\theta_{a}$ is the standard deviation of length-at-age $a$, which is modeled to be proportional to the expected length-at-age $a$, i.e.:

$$
\begin{equation*}
\theta_{a}=\beta^{z} l_{\infty}^{m / f}\left(1-e^{-\kappa\left(a-t_{0}\right)}\right) \tag{17}
\end{equation*}
$$

with $\beta^{*}$ a fixed parameter of the model, and set here to 0.20 .

### 1.1 Initial conditions

For the first year (1990) considered in the model, the stock is assumed to be at a fraction ( $\theta$ ) of its preexploitation spawning biomass, i.e.:

$$
\begin{equation*}
B_{1990}^{s p}=\theta \cdot K^{s p} \tag{18}
\end{equation*}
$$

with the starting age structure for the first year given by:

$$
\begin{equation*}
N_{1990,0}^{m, f}=\theta^{*} R \tag{19}
\end{equation*}
$$

where $R$ is the recruitment corresponding to the $K$ (the mean unexploited population size). The numbers at age for the starting population size in 1990 are then calculated as follows:

$$
\begin{equation*}
N_{1990, a}^{m / f}=N_{1990, a-1}^{m / f} e^{-\left(M_{a}+\varphi\right)} \quad \text { for } 1 \leq a \leq m \tag{20}
\end{equation*}
$$

and

$$
\begin{equation*}
N_{1990, m}^{m, f}=N_{1990, m}^{m, f} /\left[1-e^{-\left(M_{m}+\varphi\right)}\right] \quad \text { for } a=m \tag{21}
\end{equation*}
$$

where $\varphi$ is the average fishing proportion over the years immediately preceding 1990.
The value of $\varphi$ is fixed at 0.01 , and $\theta$ is an estimable parameter.
By adding a penalty to the likelihood, the value of fishing proportion in 2009, $F_{1990}$, can be set equal to any required level.

### 1.2 The von Bertalanffy Growth Function

Johnston and Butterworth (2011) reports the von Bertalanffy growth parameters (see equation 16) for each island and sex as found in the literature. Note that the data for males were from Tristan and Nightingale only, and for females data from Tristan only. Johnston and Butterworth (2011) provides suggested von Bertalanffy parameters for each island and sex, based on discussions with James Glass (pers. commn). The original growth parameter estimates were obtained from studies by Pollock and Roscoe (1977) and Pollock (1981). The tagging data centered around lobsters of carapace length 85 mm . Original model fits using the von Bertalanffy parameters as reported in Johnston (2011) did not produce satisfactory fits to catch-at-length data. It was found that by changing the $l_{\infty}$ value slightly one could greatly improve model fits. The authors thus decided to follow the following method for setting the von Bertalnaffy growth parameters:

- Since most of the tagging data centered around carapace length 85 mm , it would be assumed that the length increment for this length (i.e. 85 mm CL) would remain fixed at the value reported in the literature.
- The $l_{\infty}$ value would be allowed to be increased or decreased in order to produce better fits to the CPUE and CAL data (this was done by fixing the $l_{\infty}$ values at different values and inspecting the resultant model fits).
- The $\kappa$ value would be re-calculated for the new $l_{\infty}$ value, assuming a "pivot" through the growth increment line at 85 mm ; thus as the $l_{\infty}$ value changes, so does the $\kappa$ value, but the growth increment at 85 mm is not altered.


Figure A2: Growth function.
In Figure A2 above, the solid line shows fit to the data as reported in the literature with $l_{\infty}(1)(\operatorname{linf}(1)$ in plot above) being the estimate produced. The dotted line shows how the authors modified this line by increasing the $l_{\infty}$ value in this case, but retaining the growth increment at the pivot CL of 85 mm .

For a new $l_{\infty}$ value, $l_{\infty}(2)$, a new kappa value, $\kappa(2)$ (the slope parameter) is calculated as follows:

$$
\begin{equation*}
\kappa(2)=\frac{l_{\infty}(1)-85}{l_{\infty}(2)-85} \cdot \kappa(1) \tag{22}
\end{equation*}
$$

The table below reports the values used in the final assessments.

|  | Inaccessible and <br> Tristan | Nightingale and <br> Gough - Pollock <br> growth |
| :---: | :---: | :---: |
| $l_{\infty}^{m}(1)$ | 132.4 | 156.5 |
| $l_{\infty}^{m}(2)$ | 125 | 150 |
| $l_{\infty}^{f}(1)$ | 99.8 | 99.8 |
| $l_{\infty}^{f}(2)$ | 90 | 90 |
| $\kappa^{m}(1)$ | 0.11 | 0.066 |
| $\kappa^{f}(1)$ | 0.06 | 0.06 |
| $t_{0}^{m}$ | 0 | 0 |
| $t_{0}^{f}$ | -15 | -15 |

### 1.3 Discard Mortality

The number of lobsters that die due to discard mortality is calculated as follows:

$$
\begin{align*}
& D_{y, a, l}^{m}=d\left(\vec{N}_{y, a, l}^{m} S_{y, l}^{m, c o m m} F_{y}\right)  \tag{23}\\
& D_{y, a, l}^{f}=d\left(\vec{N}_{y, a, l}^{f} S_{y, l}^{f, c o m m} F_{y}\right) \tag{24}
\end{align*}
$$

where $D_{y, a, l}^{m / f}$ is calculated for $l<\min$, and $d$ is the value of discard mortality which is set equal to 0.1 here.

### 1.4 Biomass estimates

The model estimate of mid-year exploitable biomass for the commercial catch is given by:

$$
\begin{equation*}
B_{y}=B_{y}^{m}+B_{y}^{f} \tag{25}
\end{equation*}
$$

where

$$
\begin{align*}
& B_{y}^{f}=\sum_{a} \sum_{l \geq \min }\left[S_{y, l}^{f, \text { comm }} w_{l}^{f} \vec{N}_{y, a, l}^{f} e^{-M_{a} / 2}\right]\left[1-\left(F_{y} S_{y, l}^{f, \text { comm }} / 2\right)\right]  \tag{26}\\
& B_{y}^{m}=\sum_{a} \sum_{l \geq \min }\left[S_{y, l}^{m, \text { comm }} w_{l}^{m} \vec{N}_{y, a l}^{m} e^{-M_{a} / 2}\right]\left[1-\left(F_{y} S_{y, l}^{m, \text { comm }} / 2\right)\right] \tag{27}
\end{align*}
$$

and where
$B_{y}$ is the total (male plus female) model estimate of mid-year exploitable biomass for year $y$.

The model estimate of begin-year biomass for the biomass survey is given by:

$$
\begin{equation*}
B_{y}^{s u r v, L e g 1}=B_{y}^{m, s u r, L e g 1}+B_{y}^{f, s u r, L \text { Leg } 1} \tag{28}
\end{equation*}
$$

where

$$
\begin{align*}
& B_{y}^{f, \text { surv,Leg } 1}=\sum_{a} \sum_{l}\left[S_{y, l}^{f, s u r v} w_{l}^{f} \vec{N}_{y, a, l}^{f}\right]  \tag{29}\\
& B_{y}^{m, s u r, L \text { Leg } 1}=\sum_{a} \sum_{l}\left[S_{y, l}^{m, s u r v} w_{l}^{m} \vec{N}_{y, a, l}^{m}\right] \tag{30}
\end{align*}
$$

and where
$B_{y}^{s u r v, L e g 1}$ is the total (male plus female) model estimate of begin-year survey biomass for year $y$.

### 1.5 Commercial catch-at-length proportions

$$
\begin{align*}
& \hat{p}_{y, l}^{c o m m, m}=\frac{\sum_{a} \vec{c}_{y, a, l}^{m}}{\sum_{a} \vec{c}_{y, a, l}^{m} \vec{c}_{y, a, l}^{f}}  \tag{31}\\
& \hat{p}_{y, l}^{c o m m, f}=\frac{\sum_{a} \vec{c}_{y, a, l}^{f}}{\sum_{a} \vec{c}_{y, a, l}^{m}+\vec{c}_{y, a, l}^{f}} \tag{32}
\end{align*}
$$

where $\hat{p}_{y, l}^{\text {comm,m/f }}$ is the estimated proportion of commercial catch of $m / f$ lobsters in length class $l$ in year $y$.

### 1.6 Biomass survey catch-at-length proportions

For Leg1 we have:

$$
\begin{align*}
& \hat{p}_{y, l}^{\text {Les } 1, m}=\frac{\sum_{a} \vec{N}_{y, a, l}^{m} S_{l}^{m, s u r v}}{\sum_{a}\left[\vec{N}_{y, a, l}^{m} S_{l}^{m, s u r v}+\vec{N}_{y, a, l}^{f} S_{l}^{f, s u r v}\right]}  \tag{33}\\
& \hat{p}_{y, l}^{\text {Leg } 1, f}=\frac{\sum_{a} \vec{N}_{y, a l}^{f} S_{l}^{f, s u r v}}{\sum_{a}\left[\vec{N}_{y, a, l}^{m} S_{l}^{m, s u r v}+\vec{N}_{y, a, l}^{f} S_{l}^{f, s u r v}\right]} \tag{34}
\end{align*}
$$

where $\hat{p}_{y, l}^{\text {Leg } 1, m / f}$ is the estimated proportion of biomass survey lobsters in Leg1 of $m / f$ lobsters in length class $l$ in year $y$.

### 1.7 Commercial selectivity-at-length function

Male and female selectivities are estimated separately as follows:

$$
\begin{align*}
& S_{y, l}^{m, \text { comm }}=\frac{e^{-\left(\mu^{m}+\varepsilon_{y}^{m}\right) l}}{1+e^{-\delta^{m}\left(l-l^{m}\right)}} e^{\left(\varepsilon_{y}^{m}\right)}  \tag{35}\\
& S_{y, l}^{f, \text { comm }}=P \frac{e^{-\left(\mu^{f}+\varepsilon_{y}^{f}\right) l}}{1+e^{-\delta^{f}\left(l-l l^{f}\right)}} e^{\left(\varepsilon_{y}^{f}\right)} \tag{36}
\end{align*}
$$

Time-varying selectivity is effected by estimating different $\boldsymbol{\mu}$ values (for $m$ and $f$ separately) for the selectivity function for each sex. These " $\mu$ " values determine the shape of the descending limb of the selectivity curve.

The estimable parameters are thus:

- $l_{*}^{m / f}$,
- $\mu^{m / f}$,
- $\delta^{m / f}$,
- $P$,
- $\varepsilon_{y}^{m / f}$ (with values for each of years for which data are available) and
- $\varepsilon_{y}^{P}$ (with values for each of years for which data are available).
where

$$
\begin{align*}
& \varepsilon_{y}^{m} \sim N\left(0,\left(\sigma_{\mu}^{2}\right)\right)  \tag{37}\\
& \varepsilon_{y}^{f} \sim N\left(0,\left(\sigma_{\mu}^{2}\right)\right)  \tag{38a}\\
& \varepsilon_{y}^{P} \sim N\left(0,\left(\sigma_{P}^{2}\right)\right) \tag{38b}
\end{align*}
$$

where $\sigma_{\mu}=0.02$ and $\sigma_{P}=0.2$.

Consequently a penalty term is added to the likelihood:

$$
\begin{equation*}
-\ln L \rightarrow-\ln L+\frac{1}{2 \sigma_{\mu}^{2}} \sum_{1997}^{2019}\left[\left(\varepsilon_{y}^{m}\right)^{2}+\left(\varepsilon_{y}^{f}\right)^{2}\right] \tag{39}
\end{equation*}
$$

The selectivity functions for males are scaled so that the maximum selectivity value is 1.0 , and the female selectivity function is scaled by the multiplicative parameter $P$ so that the maximum selectivity value for females is equal to $P$.

### 1.8 Survey selectivity-at-length function

The selectivity functions for the gear used in the biomass surveys are assumed to be time invariant.
Male and female selectivities are estimated separately as follows:

$$
\begin{align*}
& S_{l}^{m, s u r v}=\frac{e^{-\mu^{m} l}}{1+e^{-\delta^{m}\left(l-l^{m}\right)}}  \tag{40}\\
& S_{l}^{f, \text { surv }}=P \frac{e^{-\mu^{f} l}}{1+e^{-\delta^{f}\left(l-l_{s}^{f}\right)}} \tag{41}
\end{align*}
$$

The estimable parameters for the survey selectivity function are thus:

- $l_{*}^{m / f}$,
- $\delta^{m / f}$,
- $\mu^{m / f}$ and
- $\quad P$ (the female scaling parameter)


## Further modifications for Gough

In order to fit the smaller size classes better a linear piecewise approach was taken where fixed values of selectivity were input.

For commercial male selectivity:
$S_{y, 40}^{m, c o m m}=0$
$S_{y, 50}^{m, \text { comm }}=0.00001$
$S_{y, 55}^{m, c o m m}=0.0001$
$S_{y, 60}^{m, c o m m}=0.0013$
$S_{y, 65}^{m, \text { comm }}=0.0014$
$S_{y, 70}^{m, \text { comm }}=0.1$
$S_{y, 75}^{m, \text { comm }}=$ from equation 35

For commercial female selectivity:
$S_{y, 40}^{f, c o m m}=0$
$S_{y, 65}^{f, \text { comm }}=0.003$

For survey female selectivity:
$S_{y, 65}^{f, \text { comm }}=0.0003$
$S_{y, 75}^{f, c o m m}=$ from equation 41
Even further modification to the commercial female selectivity function are included in the 2021 assessment, again to improve fits to the size classes $60 \mathrm{~mm}-80 \mathrm{~mm}$.

In order to smooth out the peak in the commercial female selectivity function around 70 cm , the following function is defined:
$S_{y, l}^{f, c o m m}=S_{y, l}^{f, c o m m} * F_{l}$
where
$F_{l}=1.0$ for $l \leq 60 \mathrm{~mm}$ CL
$F_{l}=1.0$ for $l \geq 80 \mathrm{~mm}$ CL
$F_{l}$ is a str line down from 1 at 60 mm to $X$ at 70 mm between lengths of 60 mm and 70 mm $F_{l}$ is a str line down up $X$ at 70 mm to 1 at 80 mm between lengths of 70 mm and 80 mm

X is a further estimable parameter.

## 2. The likelihood function

The model is fitted to CPUE, survey abundance, commercial catch-at-length (male and female separately) data and survey catch-at-length (male and female separately) data to estimate model parameters. Contributions by each of these to the negative log-likelihood (-lnL), and the various additional penalties added are as follows. For the outer islands, the model is also fitted to discard ratio data.

### 2.1 Relative abundance data (CPUE) from commercial catch

The likelihood is calculated assuming that the observed abundance index is log-normally distributed about its expected (median) value:

$$
\begin{equation*}
C P U E_{y}=q B_{y} e^{\varepsilon_{y}} \text { or } \varepsilon_{y}=\ln \left(C P U E_{y}\right)-\ln \left(q B_{y}\right) \tag{42}
\end{equation*}
$$

where
$C P U E_{y}$ is the CPUE abundance index for year $y$,
$B_{v}$ is the model estimate of mid-year exploitable biomass for year $y$ in given by equation 25 ,
$q$ is the constant of proportionality (catchability coefficient), and
$\varepsilon_{y}$ from $N\left(0,(\sigma)^{2}\right)$.

The contribution of the abundance data to the negative of the log-likelihood function (after removal of constants) is given by:

$$
\begin{equation*}
-\ln L=\sum_{y}\left[\left(\varepsilon_{y}\right)^{2} / 2(\sigma)^{2}+\ln (\sigma)\right] \tag{43}
\end{equation*}
$$

where
$\sigma$ is the residual standard deviation estimated in the fitting procedure by its maximum likelihood value:

$$
\begin{equation*}
\hat{\sigma}=\sqrt{1 / n \sum_{y}\left(\ln C P U E_{y}-\ln \hat{q} \hat{B}_{y}\right)^{2}} \tag{44}
\end{equation*}
$$

where
$n$ is the number of data points in the CPUE series, and
$q$ is the catchability coefficient, estimated by its maximum likelihood value:

$$
\begin{equation*}
\ln \hat{q}=1 / n \sum_{y}\left(\ln C P U E_{y}-\ln \hat{B}_{y}\right) \tag{43}
\end{equation*}
$$

### 2.2 Relative abundance data from the biomass survey

The likelihood is calculated assuming that the observed abundance index is log-normally distributed about its expected (median) value:

$$
\begin{equation*}
S U R_{y}^{L e g}=q^{L e g} B_{y}^{\text {sur }} e^{\varepsilon_{y}}, \quad \text { i.e. } \quad \varepsilon_{y}=\ln \left(S U R_{y}^{L e g}\right)-\ln \left(q^{L e g} B_{y}^{\text {sur }}\right) \tag{45}
\end{equation*}
$$

where
$S U R_{y}^{L e g}$ is the survey biomass abundance index for Leg1 in year $y$,
$B_{y}^{s u r}$ is the model estimate of mid-year exploitable survey biomass for year $y$ given by equation 28,
$q^{\text {Leg }}$ is the constant of proportionality (catchability coefficient) for Leg1, and
$\varepsilon_{y}$ from $N\left(0,(\sigma)^{2}\right)$.

The contribution of the biomass survey abundance data to the negative of the log-likelihood function (after removal of constants) is given by:

$$
\begin{equation*}
-\ln L=\sum_{y}\left[\left(\varepsilon_{y}\right)^{2} /\left(2\left(\left(C V_{y}^{L e g}\right)^{2}+\sigma_{a d d}^{2}\right)\right)+0 \cdot 5 \cdot \ln \left(\left(C V_{y}^{L e g}\right)^{2}+\sigma_{a d d}^{2}\right)\right] \tag{46}
\end{equation*}
$$

where
$C V_{y}^{\text {Leg }}$ is the survey sampling CV of the biomass survey in year $y$ for Leg1,
$q^{\text {Leg }}$ is the catchability coefficient for Leg1, estimated by its maximum
likelihood value:

$$
\begin{equation*}
\ln \hat{q}^{L e g}=1 / n \sum_{y}\left(\ln S U R_{y}^{L e g}-\ln \hat{B}_{y}^{s u r}\right), \text { and } \tag{47}
\end{equation*}
$$

$\sigma_{a d d}$ is an estimable parameter which reflects variance additional to the estimated survey sampling variance.

Furthermore, the -InL contribution is modified in order to prevent the model from giving too much weight to the CPUE data (i.e. fitting the CPUE data perfectly by allowing for the $\varepsilon_{y}$ values to vary sufficiently. The contribution of the abundance data to the negative of the log-likelihood function (after removal of constants) is given by:

$$
\begin{equation*}
-\ln L=\sum_{y}\left[\frac{\varepsilon_{y}^{2}}{2\left(\sigma^{2}+c^{3}\right)}+\frac{1}{2} \ln \left(\sigma^{2}+c^{2}\right)\right] \tag{48a}
\end{equation*}
$$

where
$\sigma$ is the residual CPUE standard deviation estimated in the fitting procedure by its maximum likelihood value:

$$
\begin{equation*}
\hat{\sigma}=\sqrt{1 / n \sum_{y}\left(\ln C P U E_{y}-\ln \hat{q} \hat{B}_{y}\right)^{2}} \tag{48b}
\end{equation*}
$$

and $c$ is a constant used to prevent the CPUE data receiving too much weight in the likelihood.
In order to keep the realised CPUE residual standard deviation to a reasonable value $\sim 0.10-0.15$, the following values were selected:

$$
\begin{aligned}
\varepsilon_{\mu} & =0.02 \\
c & =0.6
\end{aligned}
$$

### 2.3 Commercial catches-at-length

The following term is added to the negative log-likelihood:
$-\ln L^{\text {length }}=w_{l e n} \sum_{y} \sum_{l} \sum_{m / f}\left\lfloor\ln \left(\sigma_{l e n} / \sqrt{p_{y, l}^{\text {conm,m/f }}}\right)+p_{y, l}^{\text {conmm/m/f }}\left(\ln p_{y, l}^{\text {conm,m/f }}-\ln \hat{p}_{y, l}^{\text {conm,m/f }}\right)^{2} / 2\left(\sigma_{l e n}\right)^{2}\right\rfloor$
where
$p_{y, l}^{c o m m, m / f} \quad$ is the observed proportion of $m / f$ lobsters (by number) in length group $l$ in the commercial catch in year $y$, and
$\sigma_{l e n} \quad$ is the standard deviation associated with the length-at-age data, which is estimated in the fitting procedure by:

$$
\begin{equation*}
\hat{\sigma}_{l e n}=\sqrt{\sum_{m / f} \sum_{y} \sum_{l} p_{y, l}^{c o m m, m / f}\left(\ln p_{y, l}^{c o m m, m / f}-\ln \hat{p}_{y, l}^{c o m m, m / f}\right)^{2} / \sum_{m / f} \sum_{y} \sum_{l} 1} \tag{50}
\end{equation*}
$$

Equation (49) makes the assumption that proportion-at-length data are log-normally distributed about their model-predicted values. The associated variance is taken to be inversely proportional to $p_{y, l}^{c o m m, m / f}$ to downweight contributions from observed small proportions which will correspond to small predicted sample sizes. The value of $w_{l e n}$ is set equal to 0.1 .

### 2.4 Biomass survey catches-at-length

The following term is added to the negative log-likelihood:

$$
\begin{equation*}
-\ell \operatorname{n} L^{\mathrm{length}}=w_{l e n} \sum_{y} \sum_{l} \sum_{m / f}\left[\ln \left(\sigma_{l e n} / \sqrt{p_{y, l}^{L e g, m / f}}\right)+p_{y, l}^{L e g, m / f}\left(\ln p_{y, l}^{L e g, m / f}-\ln \hat{p}_{y, l}^{L e g, m / f}\right)^{2} / 2\left(\sigma_{l e n}\right)^{2}\right] \tag{51}
\end{equation*}
$$

where
$p_{y, l}^{\text {Leg,m/f }} \quad$ is the observed proportion of $m / f$ lobsters (by number) in length group $I$ in the biomass survey in year $y$ during Leg1 or Leg2, and
$\sigma_{l e n} \quad$ is the standard deviation associated with the length-at-age data, which is estimated in the fitting procedure by:

$$
\begin{equation*}
\hat{\sigma}_{l e n}=\sqrt{\sum_{m / f} \sum_{y} \sum_{l} p_{y, l}^{L e g, m / f}\left(\ln p_{y, l}^{L e g, m / f}-\ln \hat{p}_{y, l}^{L e g, m / f}\right)^{2} / \sum_{m / f} \sum_{y} \sum_{l} 1} \tag{52}
\end{equation*}
$$

Equation (49) makes the assumption that proportion-at-length data are log-normally distributed about their model-predicted values. The associated variance is taken to be inversely proportional to $p_{y, l}^{s u r v, m / f}$ to downweight contributions from observed small proportions which will correspond to small predicted sample sizes. The value of $w_{l e n}$ is set equal to 0.1 .

### 2.3 Discard \% (for Nightingale, Inaccessible and Gough)

The longline catch and effort databases provide information on the weight of discarded lobsters. In this document the discard \% is expressed as \% weight of discards relative to the weight of the total catch (under and over legal size) hauled. This information is incorporated into the likelihood function when fitting the assessment models to the data by including the following term:

$$
\begin{equation*}
-\ln L=-\ln L+\sum_{y}\left(\operatorname{lnD}_{\mathrm{y}}^{\mathrm{obs}}-\ln \widehat{\mathrm{D}_{\mathrm{y}}}\right)^{2} / 2 \mathrm{CV}^{2} \tag{53}
\end{equation*}
$$

where
$D_{y}^{o b s}$ is the observed discard percentage for year $y$, and
$\widehat{D}_{y} \quad$ is the model estimated value of discard percentage for year y , where
$D_{y}^{*}=\sum_{a=0}^{P} \sum_{l=1}^{\min -1}\left(w_{l}^{m} D_{y, a, l}^{m}+w_{l}^{f} D_{y, a, l}^{f}\right)$,
and
$\widehat{D}_{y}=\left[D_{y}^{*} /\left(D_{y}^{*}+C_{y}\right)\right] * 100$

The CV is set at a value of 0.6 for Inaccessible and Nightingale, and 0.8 for Gough.

### 2.4 Stock-recruitment function residuals

The assumption that these residuals are log-normally distributed (and could be serially correlated) defines a corresponding joint prior distribution. This can be equivalently regarded as a penalty function added to the log-likelihood, which for fixed serial correlation $\rho$ is given by:

$$
\begin{equation*}
-\ln L=-\ln L+\sum_{y=y}^{12}\left[\frac{\varepsilon_{y}-\rho \varepsilon_{y-1}}{\sqrt{1-\rho^{2}}}\right]^{2} / 2 \sigma_{R}^{2} \tag{56}
\end{equation*}
$$

where

$$
\begin{aligned}
& \varsigma_{y}=\rho \tau_{y-1}+\sqrt{1-\rho^{2}} \varepsilon_{y} \text { is the recruitment residual for year } y \text { (see equation 1), } \\
& \varepsilon_{y} \sim N\left(0, \sigma_{R}^{2}\right), \\
& \sigma_{R} \text { is the standard deviation of the log-residuals, which is input, } \\
& \rho \text { is their auto-correlation coefficient, and } \\
& y 1=1992 \text { and } y 2=2017 \text { here. }
\end{aligned}
$$

Note that here, $\rho$ is set equal to zero, i.e. the recruitment residuals are assumed uncorrelated, and $\sigma_{R}$ is set equal to 0.4. Recruitment residuals are estimated for years 1992 to 2017 only.

The following term is added to constrain the size of these terms (i.e. to fit to genuine difference rather than to noise) and to force the average of the residuals to equal zero:

$$
\begin{equation*}
-\ln L=-\ln L+W\left[\sum_{1992}^{2016} \frac{\varepsilon_{y}}{\sigma_{R}}\right]^{2} \tag{57}
\end{equation*}
$$

where the weighting factor $W$ is set high to ensure that the sum above ends as zero. This is to ensure that when projecting, the stock-recruitment curve used more closely reflects the past patterns of recruitment and its variability.

Future recruitment: The model estimates residuals for 1992-2017. For 2018+ recruitment is set equal to its expected values given the fitted stock-recruit relationship. The relationship itself is $R_{y}=\frac{\alpha B_{y}^{s p}}{\beta+B_{y}^{s p}} e^{\varepsilon_{y}-\sigma_{R}^{2} / 2}$ where $\varepsilon_{y} \sim N\left(0, \sigma_{R}^{2}\right) \quad$ and $\quad \sigma_{R}=0.4$. This means that the expected recruitment $E\left[R_{y}\right]=\frac{\alpha B_{y}^{s p}}{\beta+B_{y}^{s p}}$

The residuals for years 1990 and 1991 are set equal to zero.

## 3. Further Model parameters

Minimum age: Age 0.
Maximum age: $p=20$, and is taken as a plus-group.
Minimum length: 1 mm .
Maximum length: 180 mm , which is taken as a plus-group.
Mass-at-age: The mass $w_{a}^{m / f}$ of a $m / f$ lobster at age $a$ is given by:

$$
\begin{equation*}
w_{a}^{m / f}=\alpha^{m / f}\left[\hat{l}_{\infty}^{m / f}\left(1-e^{-\hat{\kappa}^{m / f}\left(a-\hat{t}_{0}^{m / f}\right)}\right) / 10\right]^{\beta^{m / f}} \tag{59}
\end{equation*}
$$

where the values assumed for the observed length-weight are:

$$
\begin{gathered}
\alpha^{m}=0.4789 \\
\alpha^{f}=0.5907 \\
\beta^{m}=3.024
\end{gathered}
$$

$$
\beta^{f}=2.9449
$$

This provides weight of in units of kgs.

## 4. The Bayesian approach

The Bayesian method entails updating prior distributions for model parameters according to the respective likelihoods of the associated population model fits to the CPUE and catch-at-length, to provide posterior distribution for these parameters and other model quantities.

The catchability coefficients $(q)$ and the standard deviations associated with the commercial CPUE and catch-at-length data ( $\sigma$ and $\sigma_{k n}$ ) are estimated in the fitting procedure by their maximum likelihood values, rather than integrating over these three parameters as well. This is considered adequately accurate given reasonably large sample sizes.

Modes of posteriors, obtained by finding the maximum of the product of the likelihood and the priors, are then estimated rather than performing a full Bayesian integration, due to the time intensiveness of the latter.

### 4.1 Priors

The following prior distributions are assumed:
$h \quad \mathrm{~N}\left(0.95, \mathrm{SD}^{2}\right)$ with $\mathrm{SD}=0.2$, where the normal distribution is truncated at $h=1$.
$l_{*}^{m / f}: \quad \mathrm{U}[1,180] \mathrm{mm}$
$\mu^{m / f} \quad \mathrm{U}[0,1]$
$\delta^{m / f} \quad \mathrm{U}[0,1]$
$P \quad \mathrm{U}[0,5]$
$\theta \quad \mathrm{U}[0,1]$
$\varsigma_{y} \quad U[-5,5]$
$\sigma_{\text {add }} \quad \mathrm{U}[0,1]$

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[^0]:    ${ }^{1}$ Cape Town Workshop held 16-18 November 2011.

