Updated 2021 horse mackerel data and assessment model description

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Summary

This document details the data available for the 2021 horse mackerel assessment. Full details of the assessment model are also provided.

Key Words: Horse mackerel, stock assessment, input data, ASPM description

Input data

The following data are available for use in the 2021 updated horse mackerel assessment:

Absolute Abundance indices: Spring survey (9 yrs) covering 1986-2016

Autumn survey (22 yrs) covering 1988-2019 (data for 2021 are available but the assessment only incorporates data up to and including 2020).

Midwater CPUE (Dessert Diamond): 2003-2020

Dual rights CPUE: 2007-2020

Catches: Demersal, midwater and pelagic for 1949-2020.

Catch-at-length data: Spring survey up to 2016, Autumn survey up to 2019

Midwater 2003-2020

These data are reported in Tables 3-5.

Model description

The full model specifications are provided below. Note the following:

The model assumes that 2014-2016 low CPUE values are a result of reduced fishing catchability. Thus

$q = q_1$ for years up to and including 2013,

$q = q_2$ for year 2014, and

$q = q_1$ for year 2017+ .

q is linearly interpolated between q_2 (in 2014) and q_1 (in 2017) to obtain the q values for 2015 and 2016 (although note that the observed 2015 CPUE value is omitted in the model fit, as decided earlier by the DWG).

Survey selectivity: the model estimates two survey selectivity functions. The “OLD” selectivity function is used for surveys conducted with the old gear, with the “NEW” selectivity function used for surveys conducted with the new gear. Table 4 indicates for which years each gear type was used. The “OLD” gear selectivity function is assumed to apply for the demersal fisheries bycatches taken. Specifications for commercial selectivities are detailed below.

The full population assessment model used for the horse mackerel assessment is described below, and provides a description of assumptions made for projecting the resource into the future.

1.1 Population Model

An age-structured production model (ASPM) is used as the underlying assessment model for the Horse Mackerel. This model is able to fit to CPUE indices as well as catch-at-length data to allow for past recruitment fluctuations to be estimated. The assessment process involves developing a model of the resource dynamics and conditioning its output to the available data by minimising a log-likelihood function. A single-stock model is used which is based largely on the assessment model by Johnston and Butterworth (2007), except that a midwater CPUE series, time-varying selectivity, catch-at-length data and recruitment fluctuations are incorporated. Important features of the model are described below, with full specifications given.

1.1.1 Dynamics

The ASPM reflects the dynamics of the resource over the period 1949-2018. The resource has been managed as a single stock (since 2001). It is assumed that the population was in equilibrium at its carrying capacity in 1949. In reality, horse mackerel catches have been taken as bycatch in other fisheries since the 1900s but these catches were recorded only from 1949, shortly after substantial development of the pelagic fishery commenced. Nevertheless, the cumulative catch before 1949 is unlikely to have been high.

Pope’s approximation to the Baranov equations is used to determine fishing mortality (Pope 1972). This assumes that all catches are taken as a pulse in the middle of the fishing season, instead of continuously throughout. The number of recruits at the start of a new year is related to the biomass of the mature component of the population (i.e. spawning biomass) of the previous year by a stock-recruitment relationship. A Beverton-Holt form is assumed. Additionally, stock-recruitment residuals that reflect natural fluctuations about expected recruitment are estimated for the years 1986-2017 (there is insufficient information content in the catch-at-length information to estimate beyond this 2017). Despite this variability in recruitment, the model assumes that at the start of the fishery in 1949, the population is stable at its unexploited equilibrium. Selectivity functions for the midwater and demersal fleets are estimated during the fitting procedure, and are assumed to have a Gaussian dependence on length. Demersal fishing selectivity is additionally assumed to vary for the “OLD” trawl gear and the “NEW” trawl gear.

1.1.2 Likelihoods

The assessment model is conditioned on survey abundance and catch-at-length data, and on commercial CPUE and catch-at-length data. Additional contributions to the negative of the (penalised) log-likelihood come from the stock-recruitment residuals and various penalty functions which are discussed below.

The midwater CPUE and demersal survey time-series are both considered to be relative indices of abundance, each proportional to the biomass available to their respective fleets at midyear. However, without any estimates of biomass in absolute terms, the model is unable to estimate catchability coefficients for these indices reliably. The autumn survey is therefore treated as an absolute index by fixing its catchability to one of three values considered to span a plausible range. The value of this catchability parameter is a key uncertainly of the model. Likelihoods are calculated by assuming that observed indices are log-normally distributed about their expected values. Although estimates of sampling variability are given for each demersal survey, the model estimates additional variance because there are likely to be other sources of variability; otherwise unrealistically high precision, and hence weight in the fitting procedure, would be accorded to these indices.

Because the assessment model is age-structured, catch-at-age estimates must be transformed into catch-at-length estimates before they can be compared to the observed catch-at-length data. This is done via an age-length matrix that is based on an input von Bertalanffy growth curve. The likelihood contributions are then calculated by comparing the model-predicted length distribution of horse mackerel catches with empirical data. Errors are assumed to be log-normally distributed.

The stock-recruitment residuals are also assumed to be log-normally distributed with no auto-correlation. Unfortunately, their variability cannot be estimated within the maximum likelihood framework used in this assessment, because the penalised likelihood function will always yield a minimum in the limit of the extent of this variability approaching zero. This issue is somewhat problematic, because recruitment fluctuations are of particular importance to the testing of pelagic MPs. While it could be dealt with by adopting a fully Bayesian methodology, it is simpler and adequate for present purposes to input the standard deviation for those residuals as a fixed value.

Finally, there are contributions to the negative log-likelihood from penalty functions. These do not correspond to any particular observed data or prior knowledge, but are instead included to discourage the optimisation from moving into unrealistic regions of parameter space, such as those resulting in negative population numbers or fishing mortality. The models presented to date for horse mackerel achieve convergence without triggering those penalty functions.

1.1.3 Parameters

Estimable parameters (42)

A complete list of the 43 parameters estimated by the model fitting procedure is given below.

K^{sp} is the pre-exploitation spawning biomass of horse mackerel;

q_{spr} is the catchability coefficient for the spring demersal survey abundance index;

ς_y is the fluctuation about the expected recruitment for year *y*, which is estimated for years 1986-2017;

μ^m is the centre of the Gaussian selectivity-at-length curve for the midwater fleet;

λ^m controls the width of the Gaussian selectivity-at-length curve for the midwater fleet;

μ^d_{y1-y2} is the centre of the Gaussian selectivity curve for the demersal fleet for years *y1 or y2*, and is estimated for periods of “OLD” gear (*y1*) and “NEW” gear (*y2*).

λ^d_{y1-y2} controls the width of the Gaussian selectivity-at-length curve for the demersal fleet for years *y1 or y2*, and is estimated for periods of “OLD” gear (*y1*) and “NEW” gear (*y2*); and

σ^s_{add} is the square root of the additional variance for the survey abundance index *s* (*s* is either *aut* for the autumn survey of *spr* for the spring survey), and reflects variability not included in the corresponding survey CVs.

q_2 is the extra amount multiplied to the midwater q_{CPUE} for the year 2014.

Input parameters

Some parameters cannot be estimated by the model, or are adequately specified by other studies and need not be estimated. They are therefore input with fixed values. The following is a list of these parameters:

q_{aut} is the catchability coefficient for the autumn demersal survey abundance index, and is assumed to be 0.75 for the Base Case (BC) model; sensitivities consider 0.5 and 1.0;

h is the “steepness” of the Beverton-Holt stock-recruitment function, and is assumed to be 0.75 for the BC model; sensitivities consider 0.6 and 0.9;

M is the natural mortality rate of horse mackerel, and is fixed at 0.3 yr^{-1}; although this choice is somewhat arbitrary (Johnston and Butterworth 2007), Horsten (1999a) found key ASPM results to be fairly robust to alternative assumptions regarding this value. Recent assessments have also examined sensitivity to the modelling of *M*.

a_m is the age-at-maturity for South African horse mackerel, and is described by a knife-edge function of age with 100% of the population being sexually mature at 3 years (Butterworth and Clark 1996; Hecht 1990);

l_a is the expected length of a fish at age *a* in centimetres, and is based on the von Bertalanffy growth function given by Equation 1 and the growth parameters reported in Table 1 below;

w_a is the weight in metric tonnes of a fish at age *a*, and is based on the length-at-age relationship described above, in combination with the mass-at-length function given by Equation 2 and the growth parameters reported in Table 1;

$S^p_{a,y1-y2}$ is the fishing selectivity for the pelagic fleet for a fish at age *a* for years *y1-y2*, and is listed in Table 2 for the periods 1949-1962, 1963-1967 and 1968+;

σ_R is the standard deviation of the stock-recruitment log-residuals, and is assumed to be equal to 0.5, which is roughly typical for a species like horse mackerel;

γ is the CV of the length distribution of horse mackerel at any given age, and is assumed to be equal to 0.09 because this value provides good fits to catch-at-length data and lies within the expected range for a species like horse mackerel; and

w_{cal} is the weighting of the catch-at-length likelihood contributions, and is fixed at 0.35 (a weighting of 1 is equivalent of being “unweighted”).

Growth

The Cape horse mackerel has a maximum reported (fork) length of 60cm and may live to more than ten years of age (Bianchi *et al*. 1999). The length-at-age relationship used in the work presented in this thesis is taken from Kerstan (pers. commn) as quoted in Horsten (1999b). This relationship takes the form of a von Bertalanffy growth curve:

$$l_a = l_\infty(1 - e^{-\kappa(a-t_0)}) \qquad (1)$$

where

l_a is the expected total length of a fish of age a in years in centimetres;

l_∞ is the asymptotic total length in centimetres;

κ , the Brody growth coefficient, is a growth rate parameter; and

t_0 is the theoretical age at which length would be zero.

The mass-at-length relationship used for Cape horse mackerel is from Naish et al. (1991). It is provided by the power model:

$$w = \alpha(l)^\beta \qquad (2)$$

where

w is the expected weight in grams of fish;

l is the total length of the fish in centimetres; and

α and β are growth parameters.

Estimates for the parameters of these growth equations are reported in Table 1. Hecht (1990) found no difference between the mean length-at-age of males and females. This provides further support for a sex-aggregated model.

Given the limited data available at present, the assessment model in unable to reliably estimate the parameters q_{aut} (autumn survey catchability) and h (stock-recruitment steepness). Hence they must be set externally. Note that q_{aut} can be thought of as a measure of the bias in the survey absolute biomass estimates. For example, a value of 0.5 means that actual biomass is twice as large as the swept-area estimate from the surveys, whereas a value of 1 would mean that these surveys provide unbiased results. h determines the productivity of the resource, with a larger h corresponding to greater productivity. The Base Case (BC) model, which has been used in assessments and MP testing since 2014 assumes q_{aut} = 0.75; h = 0.75.

1.2 Projections

Projections are simulations of the future state of a fishery given present understanding of the resource dynamics as represented by an assessment model. By providing a basis to calculate fishery performance statistics, they give means of testing candidate MPs and enable stake-holders to make informed decisions about trade-offs. In this section projections under the assumption of constant future catches for both the future pelagic catches and future midwater catches are considered. The horse mackerel resource is projected 30 years into the future. Because there are stochastic elements in the model dynamics, 1000 projections, each using different random numbers, are simulated for each future catch scenario as explained below. This allows for realistic estimates of performance statistics. Additionally, the random number generator is seeded with the same value at the start of each set of 1000 projections in order to eliminate the variability that would results from using different seeds; this allows for readier comparisons between scenarios.

Future stock-assessment residuals are drawn randomly from a normal distribution with a stand deviation of σ_R. Additionally, they are assumed to be serially correlated, with a Pearson correlation coefficient of 0.47. This value is taken from the serial correlation of the model-estimated residuals.

Future “observed” midwater CPUE and autumn demersal and pelagic survey biomass estimates are generated during projections, because these indices of abundance are potentially useful as inputs to many MPs. Realistic observation errors are added to the expected values of these abundance indices by drawing them at random from the same log-normal distribution assumed in the assessment model (Equations 26, 28. 30 and 31 below). The variance of the error distribution for the CPUE and pelagic survey indices are estimated in the assessment (Equation 32 below), while the variance for the autumn demersal survey abundance estimate is a combination of the estimated σ_{add}^{aut} (additional variance) and a CV (Equation 29 below). Future CVs are drawn randomly with replacement from historic autumn survey CVs.

Mathematical details of the ASPM

1 Dynamics

The population dynamics are described by the following equations:

$$N_{y+1,0} = R_{y+1} \tag{3}$$

$$N_{y+1,a+1} = (N_{y,a}e^{-M/2} - C_{y,a})e^{-M/2} \qquad 0 \le a \le m-2 \tag{4}$$

$$N_{y+1,m} = \left(N_{y,m}e^{-\frac{M}{2}} - C_{y,m}\right)e^{-\frac{M}{2}} + (N_{y,m-1}e^{-\frac{M}{2}} - C_{y,m-1})e^{-M/2} \tag{5}$$

where

$N_{y,a}$ is the number of horse mackerel of age *a* at the start of year *y*;

$C_{y,a}$ is the total number of horse mackerel of age *a* taken in year *y* by the pelagic, midwater and demersal fleets combined;

R_y is the number of recruits (0-year olds) at the start of year *y*;

M is the natural mortality rate for horse mackerel; and

m is the minimum age within the plus-group and is set here to ten years old.

The approximation of the fishery as a pulse catch in the middle of the season is considered of sufficient accuracy for present purposes. Note that the model also assumes that recruitment to the population occurs at the start of a new year (Equation 3), even though in reality there are two pawning peaks roughly two months apart.

The total number of horse mackerel of age a caught each year is given by:

$$C_{y,a} = \sum_f C_{y,a}^f \tag{6}$$

where *f* indicates the fishery concerned and in this case is either *p* for pelagic, *d* for demersal or *m* for midwater.

The annual catch by mass for fleet *f* is given by:

$$C_y^f = \sum_{a=0}^m w_{y,a}^f C_{y,a}^f$$

$$= \sum_{a=0}^m w_{y,a}^f S_{y,a}^f F_y^f N_{y,a}e^{-M/2} \tag{7}$$

where

$S_{y,a}^f$ is the fishing selectivity-at-age for fleet *f* for fish of age *a* in year *y*;

F_y^f is the fleet-specific fishing mortality for a fully selected age class in year *y*; and

$w_{y,a}^f$ is the effective weight of a horse mackerel of age *a* for fleet *f* in year *y*.

Fishing selectivity for the pelagic fleet is described by selectivity-at-age functions which are input; therefore that fleet's effective weight-at-age (in gm) is simply given by a combination of the length-at-age (in cm) and weight-at-length relationships discussed above:

$$l_a = 54.56\left[1 - e^{-0.183(a+0.654)}\right] \quad (8)$$

$$w_a^p = 0.0078 l_{a+1/2}^{3.011} \mathrm{x} 10^{-6} \quad (9)$$

Because the fishing selectivities of the midwater and demersal fleets are modelled by selectivity-at-length functions, their effective weights-at-age must be calculated differently:

$$w_{y,a}^f = \frac{\sum_l w_l S_{y,l}^f A_{l,a}}{\sum_l S_{y,l}^f A_{l,a}} \quad (10)$$

where

w_l is the weight of a horse mackerel of length l (Equation 2);

$S_{y,l}^f$ is the fishing selectivity for fleet f for fish of length l in year y; and

$A_{l,a}$ is the age-length key, which gives the proportion of fish of age *a* that are of length *l* (detailed later in Equation 18).

Note that fishing selectivity for the midwater fleet is assumed to be time-invariant; therefore the *y* subscript may be dropped when determining the effective weight-at-age for that fleet.

The fleet-specific exploitable component of abundance is taken to be given by exploitable biomass at midyear:

$$B_y^f = \sum_{a=0}^{m} w_{y,a}^f S_{y,a}^f N_{y,a}\, e^{-M/2} \quad (11)$$

or in terms of numbers of individuals:

$$N_y^f = \sum_{a=0}^{m} S_{y,a}^f N_{y,a}\, e^{-M/2} \quad (12)$$

The proportion of the resource harvested each year by fleet *y* is therefore given by:

$$F_y^f = \frac{C_y^f}{B_y^f} \quad (13)$$

and

$$C_{y,a}^f = S_{y,a}^f F_y^f N_{y,a} e^{-M/2} \quad (14)$$

Note that in terms of Equations 13 and 14 the model assumes the same fishing selectivity for the commercial demersal fleet and both demersal surveys. This simplifying assumption has been made because there are no catch-at-length data available to estimate selectivity functions for the commercial fleet.

Fishing selectivities

Selectivity-at-age for the pelagic fleet is input and assumed to change with time. The same values are used as for the 2007 assessment model (Johnston and Butterworth 2007). Essentially there is one selectivity function for the pre-1993 period and another for the post-1967 period, while for the period between (1963-1967) the average of those two selectivity functions is used.

In contrast, selectivity-at-length is estimated for both the midwater (time invariant) and demersal fleets ("OLD" and "NEW" selectivity functions). These are assumed to have a Gaussian form with length:

$$S_{y,l}^{f} = e^{-\left(l-\mu_l^f\right)^2/2(\lambda_y^f)^2} \quad \text{if } l_{min}^f \leq l \leq l_{max}^f \text{ or}$$

$$= 0 \qquad \text{otherwise} \qquad (15)$$

where

μ_y^f is an estimated selectivity parameter that determines the centre of the Gaussian for fleet *f* in year *y*;

$\lambda_y f$ is an estimated parameter that determines the width of the Gaussian for fleet *f* in year *y*;

l_{min}^f is a fixed selectivity parameter that determines the smallest length class with non-zero selectivity for fleet *f*, and is set equal to 10cm for both the demersal or the midwater fleets, and

l_{max}^f is a fixed selectivity parameter that determines the largest length class with non-zero selectivity for fleet *f*, and is set equal to 50cm or 60cm for the demersal or midwater fleets respectively.

Note again that the y subscript may be dropped when dealing with selectivity for the midwater fleet because it is time-invariant. Selectivity-at-length is then normalised according to:

$$S_{y,l}^f \rightarrow S_{y,l}^{*,f} = \frac{S_{y,l}^f}{\sum_{l'=l_1}^{l_2} \frac{S_{y,l'}^f}{l_2 - l_1 + 1}} \qquad (16)$$

In other words, the selectivity function is scaled by the inverse of its average value over a certain length range. l_1 and l_2 are the same for both midwater and the demersal fleets and are set equal to 10cm and 40cm respectively.

Because the model is age-structured, selectivity-at-length must be transformed into selectivity-at-age using an age-length relationship:

$$S^f_{y,a} = \sum_l \ A_{l,a} S^f_{y,l} \tag{17}$$

It is assumed that the length distribution for horse mackerel of age *a* is described by a normal distribution with mean which is given by the von Bertalanffy growth curve input, and with a standard deviation that is proportional to this mean. Consequently, with length classes of 1cm, $A_{l,a}$ is computed according to:

$$A_{l,a} = \frac{1}{2}[\mathrm{erf}\left(\frac{l+0.5-l_{a+0.5}}{\sqrt{2}(\gamma l_{a+0.5})}\right) - \mathrm{erf}\left(\frac{l-0.5-l_{a+0.5}}{\sqrt{2}(\gamma l_{a+0.5})}\right)] \tag{18}$$

where

erf is the error function;

$l_{a+0.5}$ is the expected midyear length for a horse mackerel of age a, which is calculated using the input von Bertalanffy growth curve given by Equation 1; and

γ is the CV of the length-at-age distribution, which is fixed at 0.9.

Stock-recruitment relationship

The spawning biomass in year y is given by:

$$B^{sp}_y = \sum_{a=a_m}^{m} w_a N_{y,a} \tag{19}$$

where

a_m is the age corresponding to 100% sexual maturity, which is assumed here to be described by a knife-edge function of age; and

w_a is the mass of a horse mackerel of age *a* at the start of the year.

The number of recruits at the start of fishing year y is related to the spawner stock size by a Beverton-Holt stock-recruitment relationship:

$$R\left(B^{sp}_y\right) = \frac{\alpha B^{sp}_y}{\beta + B^{sp}_y} e^{\varsigma_y} \tag{20}$$

where

α and β are stock-recruitment parameters; and

ς_y are stock-recruitment residuals reflecting fluctuations about expected recruitment in year *y*.

In order to work with estimable parameters that are more biologically meaningful than α and β, the stock-recruitment relationship is re-parametersied in terms of pre-exploitation equilibrium spawning biomass, K^{sp}, and the steepness of the stock-recruitment relationship, *h*, where steepness is the fraction of pristine recruitment, R_0, that results when spawning biomass drops to 20% of its pristine level:

$$hR_0 = R(0.2K^{sp}) \tag{21}$$

from which it follows that:

$$h = \frac{0.2(\beta+K^{sp})}{\beta+0.2K^{sp}} \tag{22}$$

and hence:

$$\alpha = \frac{4hR_0}{5h-1} \tag{23}$$

and

$$\beta = \frac{K^{sp(1-h)}}{5h-1} \tag{24}$$

Given a value for the pre-exploitation spawning biomass K^{sp} of horse mackerel, together with the assumption of an initial equilibrium age-structure, pristine recruitment can be determined from:

$$R_0 = \frac{K_{sp}}{[\sum_{a=a_m}^{m-1} w_a e^{-aM} + w_m e^{-mM}/(1-e^{-M})]} \tag{25}$$

2 Likelihood functions

The model is fitted to three biomass indices and three sets of catch-at-length data. Stock recruitment residuals also contribute to the penalised negative log-likelihood that is minimised in the fitting process.

Abundance indices

The assessment model is ordinarily fitted to three abundance indices: spring and autumn demersal biomass estimates, and a commercial midwater CPUE series. The associated likelihood contribution are calculated by assuming that the observed abundance index is log-normally distributed about its expected value:

$$I_y^s = \widehat{I}_y^s e^{\epsilon_y^s} \text{ or } \epsilon_y^s = \ln(I_y^s) - \ln(\hat{I}_y^s) \tag{26}$$

where

s indicates the abundance index concerned and is either *aut* for the autumn survey, or *spr* for the

spring survey, *cpue* for CPUE or *pel* for the pelagic index;

I_y^s is the observed value of index *s* in year *y*;

$\hat{I}_y^s$ is the model predicted value of *s* in year *y*.

The negative of the log-likelihood function (after removal of the constant) is then given by:

$$-lnL = \sum_s \sum_y [ln\sigma_y^s + \left(\epsilon_y^s\right)^2 / 2(\sigma_y^s)^2] \quad (27)$$

The spring and autumn demersal survey biomass estimates are assumed to reflect demersal exploitable biomass:

$$\hat{I}_y^s = q_s B_y^d \quad (28)$$

where q_s is the catchability coefficient corresponding to index *s*. Note that the same demersal exploitable biomass B_y^d is used to fit both the autumn and spring demersal surveys even though they occur several months apart. Because a mid-year pulse catch assumption is made (Equation 13), this exploitable biomass does not account for fishing mortality that may occur between the surveys. For these series, reliable estimates of sampling variability and additional variance are available; therefore the standard deviations are calculated according to the following formula:

$$\sigma_y^s = \sqrt{\ln\left[1 + (CV_y^s)^2\right] + (\sigma_{add}^s)^2} \quad (29)$$

where

CV_y^s is the CV for survey s in year y, which is given in Table A2, and

σ_{add}^s is the model estimated additional variance for survey abundance index s.

The midwater CPUE index is assumed to reflect the midwater exploitable biomass:

$$\hat{I}_y^{cpue} = q_{cpue} B_y^m \quad (30)$$

and the pelagic hydro-accoustic survey index from November of year *y* is assumed to reflect recruitment in year y+1:

$$\hat{I}_y^{pel} = q_{pel} R_{y+1} \quad (31)$$

Reliable estimates of CVs and catchability are unavailable for the CPUE and pelagic abundance indexes. Therefore, they are set to their maximum likelihood estimates:

$$\sigma^s = \sqrt{1/n \sum_y (\epsilon_y^s)^2} \quad (32)$$

$$lnq_s = 1/n \sum_y \epsilon_y^s \quad (33)$$

For the midwater CPUE index:

$q_{cpue} = q_1$ for years up to and including 2013,

$q_{cpue} = q_2$ for year 2014,

$q_{cpue} = q_1$ for year 2017+ ,

q_{cpue} is linearly interpolated between q_2 (in 2014) and q_1 (in 2017) to obtain the q_{cpue} values for 2015 and 2016 (although note that the observed 2015 CPUE value is omitted in the model fit, as decided earlier by the DWG).

This model thus assumes that recent (2014-2016) low CPUE values are a result of reduced fishing catchability.

Catch-at-length

Model estimated catch-at-length proportions are fitted to spring and autumn demersal survey length-frequency data, and commercial midwater length frequency data.

Catch-at-age estimates (Equation 14) are transformed into catch-at-length estimates using the age-length relationship $A_{l,a}$ (Equation 18):

$$C_{y,l}^{f} = \sum_{a=0}^{m} A_{l,a} C_{y,a}^{f} \qquad (34)$$

where $C_{y,l}^{f}$ is the total number of horse mackerel of length *l* caught in year *y*.

The contribution of catch-at-length data to the negative log-likelihood function is then given by:

$$-lnL = w_{cal} \sum_s \sum_y \sum_l \left[ln\sigma_{cal}^{s} + \left(\sqrt{p_{y,l}^{s}} - \sqrt{\hat{p}_{y,l}^{s}}\right)^2 / 2(\sigma_{cal}^{s})^2 \right] \qquad (35)$$

where

w_{cal} is a weighting for this likelihood contribution, and is fixed at 0.35;

$p_{y,l}^{s}$ is the observed proportion of fish caught in year *y* that are of length *l* for dataset *s*;

$\hat{p}_{y,l}^{s}$ is equal to $C_{y,l}^{f} / \sum_l C_{y,l}^{f}$ and is the model predicted proportion of fish caught in year *y* that are of length *l* in dataset *s*, where *f* is the appropriate fleet; and

σ_{cal}^{s} is the standard deviation associated with catch-at-length dataset *s*, which is estimated in the fitting procedure by:

$$\sigma_{cal}^{s} = \sqrt{\sum_y \sum_l \left(\sqrt{p_{y,l}^{s}} - \sqrt{\hat{p}_{y,l}^{s}}\right)^2 / \sum_y \sum_l 1} \qquad (36)$$

Note that allowance is made for a minus group (fish smaller than 10 cm) and a plus group (fish 46 cm and larger). Length classes are specified with intervals of 5 cm.

Stock-recruitment residuals

It is assumed that these residuals are log-normally distributed and are not serially correlated. Therefore, their contribution to the penalised negative log-likelihood is given by:

$$-lnL = \sum_y \frac{\varsigma_y^2}{2\sigma_R^2} \qquad (37)$$

where

ς_y is the estimated stock-recruitment residual for year y; and

σ_R is the input standard deviation of the log-residuals, which is assumed to be equal to 0.5.

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Table 1: Parameter values for the von Bertalanffy growth curve (Equation 1) and mass-at-length relationship (Equation 2) for Cape horse mackerel. Values reported are taken from Kerstan *pers. commn*) as quoted in Horsten (1999b) and from Naish *et al.* (1991) respectively.

Parameter	Value
l_∞ (cm)	54.56
κ (yr^{-1})	0.183
t_0 (yr)	-0.654
α (g/cm^β)	0.0078
β	3.011

Table 2: Selectivity-at-age vectors assumed for the pelagic fleet over three different periods (Johnston and Butterworth 2007).

	Period		
Age (yr)	**1949-1962**	**1963-1967**	**1968+**
0	0.00	0.14	0.28
1	0.00	0.50	1.00
2	0.30	0.40	0.50
3	1.00	0.50	0.00
4	0.50	0.25	0.00
5	0.50	0.25	0.00
6	0.25	0.13	0.00
7	0.00	0.00	0.00
8	0.00	0.00	0.00
9	0.00	0.00	0.00
10+	0.00	0.00	0.00

Table 3: Horse Mackerel catch data for the three different fleets (values in MT) (data provided by DEFF).

Year	Pelagic catch	Demersal catch	Midwater catch
1949	3360	0.00001	0.00001
1950	49900	445	0.00001
1951	98900	1105	0.00001
1952	102600	1226	0.00001
1953	85200	1456	0.00001
1954	118100	2550	0.00001
1955	78800	1926	0.00001
1956	45800	1334	0.00001
1957	84600	959	0.00001
1958	56400	2073	0.00001
1959	17700	2075	0.00001
1960	62900	3712	0.00001
1961	38900	3627	0.00001
1962	66700	3079	0.00001
1963	23300	1401	0.00001
1964	24400	9522	0.00001
1965	55000	7017	0.00001
1966	26300	7596	0.00001
1967	8800	6189	0.00001
1968	1400	9116	0.00001
1969	26800	12252	0.00001
1970	7900	17872	0.00001
1971	2200	33329	0.00001
1972	1300	20560	0.00001
1973	1600	33900	0.00001
1974	2500	38391	0.00001
1975	1600	55459	0.00001
1976	400	50981	0.00001
1977	1900	116400	0.00001
1978	3600	37290	0.00001
1979	4300	53584.5	0.00001
1980	400	39187.5	0.00001
1981	6100	41215	0.00001
1982	1100	32176	0.00001
1983	2100	38332	0.00001
1984	2800	37969	0.00001
1985	700	27278	0.00001
1986	500	31378	0.00001
1987	2834	38571	0.00001
1988	6403	41482	0.00001
1989	25872	58205.5	0.00001
1990	7645	56721.3	0.00001
1992	2057	37207.53	0.00001
1993	11651	35998	0.00001
1994	8207	20029.5	0.00001
1995	1986	10790	0.00001
1996	18920	31846	0.00001
1997	12654	34670.5	0.00001
1998	26680	36278.8	15769.8
1999	2057	21579.73	2160.77
2000	4503	9228.977	15375.74
2001	915	8813.736	19220.38
2002	8148	4863.111	11098.47
2003	1012	3562.168	25290.98
2004	2048	4933.367	27154.31
2005	5627	5280.164	29005.21
2006	4824	4132.990	18068.35
2007	1903	4811.698	24251.18
2008	2280	4449.295	23774.56
2009	2087	4128.813	29021.42
2010	4385	5595.850	23479.62
2011	10990	5148.755	28828.99
2012	2199	4941.442	22578.83
2013	596	2695.003	28416.75
2014	2760	3087.010	10052.97
2015	2041	4747.106	7975.59
2016	1601	5230.374	11612.69
2017	1415	5703.439	17545.20
2018	948	4527.709	22827.79
2019	1082	4720.275	16498.21
2020	2174	4300.648	19710.07

Table 4a: GLM standardised CPUE (for the *Desert Diamond*), the Dual rights vessels' CPUE and survey abundance estimates for South African horse mackerel for the period 1986-2018. Data were provided by Coetzee, Fairweather and Singh (DEFF, *pers. commn).*

Year	Desert Diamond CPUE	Dual rights CPUE	Autumn demersal survey		Spring demersal survey	
			Biomass (KT)	CV	Biomass (KT)	CV
1986					97.36	0.13
1987					332.97	0.14
1988			152.27	0.31		
1989						
1990						
1991			325.43	0.22		
1992			385.11	0.23		
1993			399.99	0.20		
1994			315.65	0.27		
1995			179.76	0.24		
1996			252.58	0.23		
1997			240.33	0.26		
1998						
1999			300.23	0.24		
2000						
2001					316.72	0.18
2002						
2003	0.607		127.20	0.24	231.36*	0.20*
2004	0.473		192.59*	0.32*	366.50*	0.19*
2005	0.718		170.05*	0.21*		
2006	0.871		326.29	0.19	350.28	0.19
2007	1.258	0.994	215.56*	0.41*	473.22*	0.19*
2008	0.791	2.202	269.11*	0.27*	300.00*	0.17*
2009	0.835	4.820	308.36*	0.24*		
2010	1.036	4.254	267.93	0.38		
2011	1.574	6.098	193.84*	0.23*		
2012	0.677	3.854				
2013	1.097	4.432				
2014	0.209	2.311	512.54*	0.28*		
2015	-	3.381	161.18*	0.17*		
2016	0.582	2.960	103.99*	0.43*	153.32*	0.25*
2017	0.861	3.138				
2018	1.729	6.735				
2019	1.334	3.807	80.68*	0.49*		
2020	2.346	6.209				
2021			*204.47*#*	*0.34*		

*These values correspond to surveys that used the new trawl net, which was introduced in September 2003.
Not used in 2021 assessment.

Table 4b: The BC and alternate DR CPUE series (provided by Fairweather).

	BC DR CPUE	Alternate DR CPUE **(updated cpue excl records >20 tons/hr)[Used in Alt1 model]**
2011	6.098	4.818
2012	3.854	3.734
2013	4.432	3.827
2014	2.311	2.311
2015	3.381	3.157
2016	2.960	2.960
2017	3.138	2.799
2018	6.735	4.565
2019	3.807	3.807
2020	6.209	5.687

Table 4c: The West Coast Summer survey series (used in the Alt2 model).

1985	9 962
1986	4 569
1987	12 297
1988	24 393
1990	49 623
1991	38 188
1992	64 045
1993	134 552
1994	140 392
1995	112 204
1996	305 283
1997	72 424
1999	99 184
2002	83 818
2003	31 890
2004	78 396
2005	13 673
2006	15 929
2007	56 692
2008	41 627
2009	60 099
2010	40 556
2011	119 199
2012	16 593
2013	209 130
2014	162 606
2015	48 800
2016	15 500
2017	58 556
2019	5 083
2020	87 592

Table 5a: Spring demersal survey catch-at-length for South African horse mackerel (shown as proportions of numbers each year) as used in the assessment model. Provided by Fairweather (DEFF, *pers. commn*).

	Total length (cm)								
Year	0–10	10–15	15–20	20–25	25–30	30–35	35–40	40–45	45+
1986	0.0000	0.0000	0.0020	0.0900	0.2380	0.1640	0.1690	0.2310	0.1050
1987	0.0000	0.0000	0.1160	0.2230	0.1600	0.2060	0.1240	0.1290	0.0430
2001	0.0020	0.0150	0.3750	0.2550	0.1240	0.1360	0.0750	0.0150	0.0040
2003	0.0000	0.0500	0.0680	0.3760	0.3670	0.0910	0.0400	0.0080	0.0010
2004	0.0010	0.2380	0.2560	0.1610	0.2260	0.0740	0.0350	0.0080	0.0010
2006	0.0080	0.2670	0.2430	0.2880	0.1440	0.0410	0.0080	0.0010	0.0000
2007	0.0000	0.2230	0.6340	0.0950	0.0440	0.0030	0.0010	0.0000	0.0000
2008	0.0010	0.0270	0.4580	0.4290	0.0680	0.0100	0.0050	0.0020	0.0000
2016	0.0001	0.0263	0.2914	0.5157	0.1325	0.0223	0.0099	0.0008	0.0010

Table 5b: **Autumn** demersal survey catch-at-length for South African horse mackerel (shown as proportions of numbers each year) as used in the assessment models. Provided by Fairweather (DEFF, *pers. commn*).

	Total length (cm)								
Year	**0-10**	**10-15**	**15-20**	**20-25**	**25-30**	**30-35**	**35-40**	**40-45**	**45+**
1988	0.0003	0.0291	0.0721	0.0145	0.0544	0.1161	0.1507	0.3460	0.2168
1992	0.0004	0.0827	0.0699	0.0824	0.2462	0.1756	0.0913	0.1186	0.1329
1993	0.0001	0.0901	0.1445	0.1032	0.1639	0.1599	0.1485	0.1350	0.0548
1994	0.0000	0.0530	0.1251	0.1396	0.2854	0.2557	0.0542	0.0491	0.0377
1995	0.0002	0.0024	0.1043	0.1206	0.2929	0.2311	0.1225	0.0801	0.0458
1996	0.0000	0.0002	0.0095	0.0169	0.2719	0.3405	0.2708	0.0719	0.0182
1997	0.0006	0.0046	0.0484	0.0217	0.3253	0.3092	0.1682	0.0828	0.0393
1999	0.0002	0.0078	0.1208	0.0735	0.0941	0.4209	0.1487	0.0925	0.0415
2003	0.0003	0.0041	0.1250	0.2697	0.2297	0.1186	0.1762	0.0656	0.0108
2004	0.0159	0.0959	0.1499	0.1165	0.2408	0.1726	0.1465	0.0575	0.0043
2005	0.0004	0.1303	0.1237	0.1420	0.2413	0.1732	0.1474	0.0404	0.0012
2006	0.0013	0.0466	0.1615	0.2348	0.2054	0.1492	0.1608	0.0375	0.0029
2007	0.0195	0.2059	0.2433	0.2541	0.1372	0.0571	0.0497	0.0316	0.0014
2008	0.0007	0.0732	0.2967	0.3771	0.1595	0.0239	0.0147	0.0489	0.0054
2009	0.0001	0.0186	0.0642	0.4922	0.2855	0.0609	0.0392	0.0367	0.0026
2010	0.0106	0.1629	0.1055	0.1858	0.2857	0.0945	0.0414	0.0963	0.0172
2011	0.0510	0.3606	0.1140	0.1455	0.2313	0.0396	0.0183	0.0346	0.0052
2014	0.0011	0.2538	0.3791	0.3062	0.0410	0.0132	0.0043	0.0007	0.0005
2015	0.0003	0.0550	0.3614	0.4436	0.0902	0.0350	0.0078	0.0023	0.0044
2016	0.0000	0.0678	0.1958	0.3441	0.1749	0.1353	0.0490	0.0313	0.0017
2019	0.0055	0.1768	0.0689	0.2033	0.2306	0.2186	0.0776	0.0113	0.0073
2021#	*0.0004*	*0.3523*	*0.1264*	*0.0701*	*0.2285*	*0.1526*	*0.0536*	*0.0079*	*0.0084*

not used in 2021 assessment.

Table 5c: Mid-water catch-at-length data (taken from Desert Diamond).

	Total length (cm)								
Year	**prop_1-10**	**prop_11-15**	**prop_16-20**	**prop_21-25**	**prop_26-30**	**prop_31-35**	**prop_36-40**	**prop_41-45**	**prop_46+**
2003	0	0	0	0.0010	0.1350	0.2560	0.5050	0.1020	0.0010
2004	0	0	0	0.0120	0.2410	0.3820	0.3280	0.0360	0.0010
2005	0	0	0.0040	0.0790	0.2880	0.3880	0.1900	0.0350	0.0160
2006	0	0	0.0060	0.1130	0.3390	0.4030	0.1260	0.0100	0.0030
2007	0	0	0.0030	0.0900	0.2930	0.3590	0.1870	0.0540	0.0140
2008	0	0.0010	0.0430	0.2560	0.3280	0.2460	0.1110	0.0140	0.0010
2009	0	0	0.0010	0.0880	0.3860	0.3180	0.1700	0.0340	0.0020
2010	0	0	0.0180	0.2200	0.3780	0.2550	0.1000	0.0260	0.0030
2011	0	0	0.0052	0.0484	0.3976	0.1928	0.1262	0.1068	0.123
2012	0	0	0.1175	0.1337	0.3229	0.2901	0.1027	0.0306	0.0024
2013	0	0.0001	0.4181	0.2915	0.0893	0.1555	0.0395	0.0047	0.0013
2014	0	0	0.0002	0.0414	0.1093	0.5491	0.2703	0.0273	0.0024
2016	0	0	0.001	0.1707	0.5813	0.1906	0.043	0.0111	0.0022
2017	0	0	0.0004	0.1868	0.571	0.204	0.0269	0.0089	0.0019
2018	0	0	0.0003	0.0818	0.4975	0.3481	0.0585	0.0096	0.0042
2019	0	0	0.0017	0.0982	0.367	0.4282	0.0909	0.0116	0.0023
2020	0	0	0	0.0443	0.5085	0.371	0.0701	0.0049	0.0012

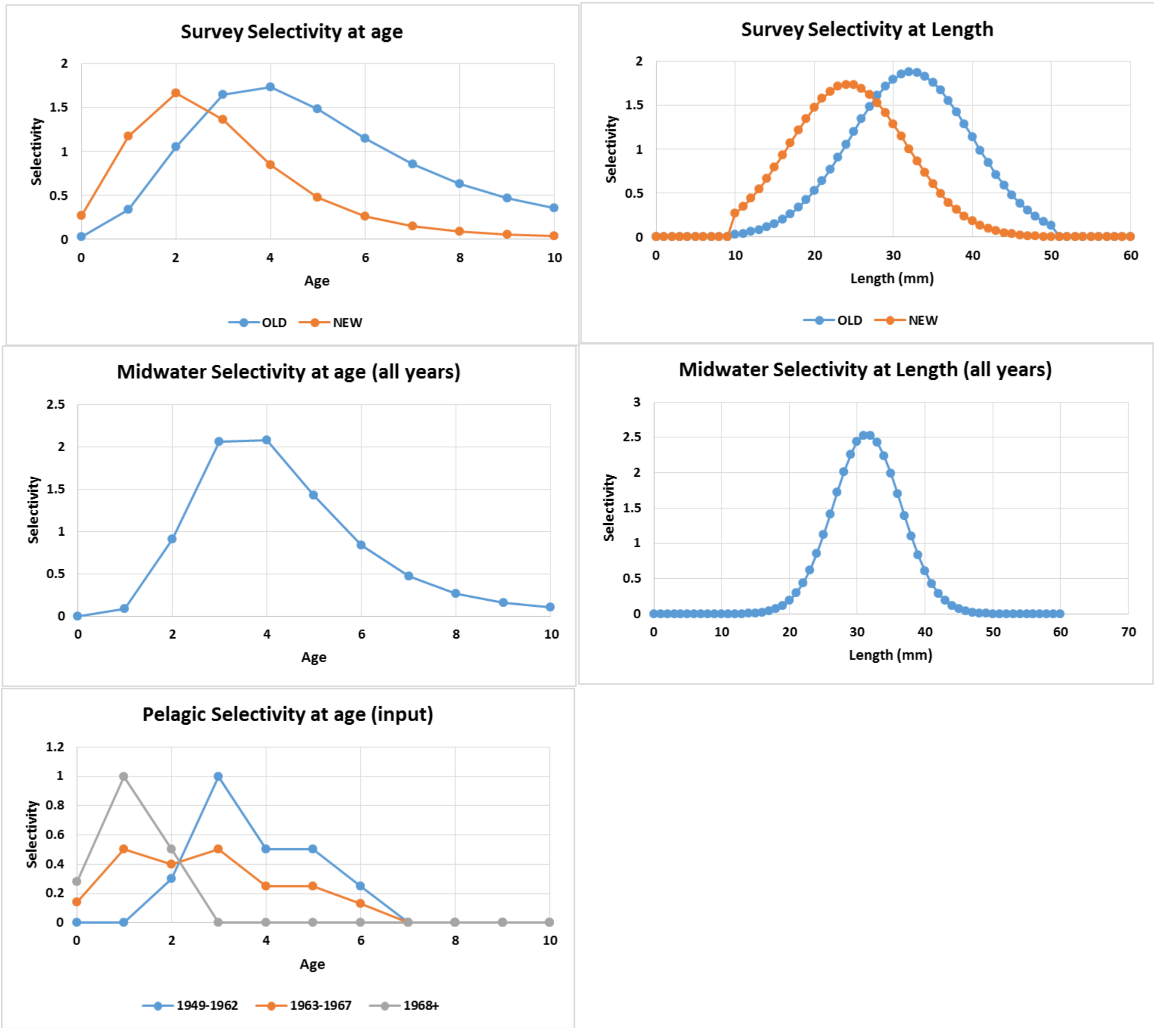

Figure 2: 2020 BC model selectivity functions. The old gear (OLD) survey selectivity plot applies also to the demersal bycatch. Demersal and midwater selectivities are estimated, and the pelagic selectivities input. The values estimated (or input for the pelagic fleet) correspond to the BC assessment.